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O r i g i n a l i n v e s t i g a t i o n s / c o m m e n t a r i e s

New Tactic of Factorization with Triangular Fuzzy Numbers

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Abstract. In this paper, three types of milk products Yellow Magic, Cow Milk, and Violet are taken into consideration for fuzzy data based on production as a fuzzy function on a discrete set of point wise low-fat milk, skim milk, and organic milk according to the age factor like Childhood, Adult and Senior citizens. In order to demonstrate the usefulness and viability of the suggested method, a novel methodology is used in this work. Triangular fuzzy numbers are used to explain the process.

Keywords: Factorization, Fuzzy Numbers, Triangular Fuzzy Numbers, Triangularization, Upper Triangular, Lower Triangular.

Introduction

Factorization is the process of dividing a number or polynomial into many factors of other polynomials, which when multiplied together yield the original number. To determine the elements that make up an integer, use the factorization formula. An entity, or set of entities, can be combined with a number, matrix, or polynomial to create a new product that, when multiplied together, returns the original number. Factorization is the name given to this technique. Using the factorization formula, a large number is divided into smaller numbers, or factors. A number is a factor if it can divide an integer by itself equally and without leaving a residue.

In order to address the linear systems coming from the augmented linear system, a group of upper and lower triangular (ULT) splitting iteration algorithms has been developed. Additionally, for some rare situations, the related convergence factors are offered in the iteration matrix of these ULT algorithms together with the hexagonal fuzzy numbers. Additionally, the effectiveness and viability of the offered solutions are evaluated, and the theoretical analysis is tested using numerical data. Experimental results show that these ULT approaches greatly outperform conventional methods in terms of numerical performance.

In numerical analysis and linear algebra, the lower-upper (LU) decomposition or factorization factors matrices as the union of lower and upper triangular matrices. The product might also contain a permutation matrix. The matrix equivalent of Gaussian elimination is LU decomposition. In addition to being a necessary step for determining a matrix's determinant or inverting matrices, LU decomposition is also frequently utilized by computers to resolve square systems of linear equations. The LU decomposition was initially suggested by Polish astronomer Tadeusz Banachiewicz in 1938. James R. Bunch and John Hopcroft presented triangular factorization and inversion by quick matrix multiplication in their paper [1]. The authors of Necessary and Sufficient Conditions for Existence of the LU Factorization, Okunev Pavel and Johnson Charles R. [6] proposed on matrix factorization and efficient least squares solution in Astronomy and Astrophysics Supplement Series. In this paper, some preliminaries presented in section 2. Section 3, illustrates the new approach with numerical examples. Conclusions are discussed in section 4.

Preliminary concepts

Definition 2.1. The characteristic function $\mu_{\tilde{A}}$ of a crisp set A $\subset$ X assigns a value either 0 or 1 to each individual in the universal set X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}}$: X → [0,1]. The assigned value indicates the membership function and the set $\widetilde{A}$ = {(x, $\mu_{\tilde{A}}(x)$); xϵX} defined by $\mu_{\tilde{A}}(x)$ for xϵX is called fuzzy set.

Definition 2.2. A ranking function is a function $\Re$: F(R) → R. Where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\widetilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number then $\Re(\widetilde{A}) = \frac{a_1+a_2+a_3}{3}$

Definition 2.3 A fuzzy number $\tilde{A}$= (C_{11}, C_{12}, C_{13}) is said to be triangular fuzzy number. If its membership function is given by

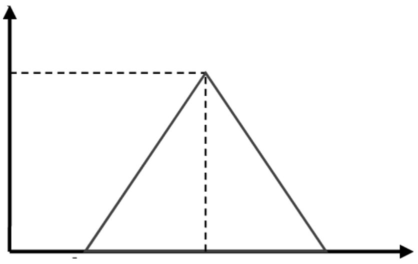

Figure 1. Representatin of Triangular Fuzzy numbers.

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{C11-x}{C12}, & C_{11} - C_{12} \le x < C_{11}, \quad C_{12} > 0 \\ 1 - \frac{x-C11}{C13}, & C_{11} \le x < C_{11} + C_{13}, \quad C_{13} > 0 \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.4

An effective approach for ordering the elements of F(R) is also to define a ranking function $\Re$:F(R) → R which maps each fuzzy number into the real line, where a natural order exists. We define orders on F(R) by:

$\tilde{a} \ge \tilde{b}$ if and only if R ($\tilde{a}$) ≥ R ($\tilde{b}$)

$\tilde{a} \le \tilde{b}$ if and only if R ($\tilde{a}$) ≤ R ($\tilde{b}$)

$\tilde{a} = \tilde{b}$ if and only if R ($\tilde{a}$) = R ($\tilde{b}$)

Definition 2.5

Low-fat milk contains less than 0-0.5%, 1%, 1.5% or 2% fat content. It has the same great health benefits and contains the same nutrients as full cream milk. Low fat milk naturally contains slightly more calcium than full cream.

Definition 2.6

Skim milk is made when all the milk fat is removed from whole milk. It contains around 0.1% fat. Organic milk is produced by cows reared under organic farming standards. These cows hormones and are fed only organic feed. Skim milk contains both the lowest amount of calories and fat compared to other dairy milks. It's included to fresh, powdered and long life milk varieties also.

Definition 2.7

Organic milk comes from cows that have never been treated with antibiotics. Organic milk cow does not have supplemental hormones. Organic milk cows receive at least 30% of nutrition from pasture during grazing season. Organic milk cows are fed organic and graze the organically managed pasture. It contains 2%, 1% and 0% of fat.

Proposed Method

In numerical analysis lower and upper decomposition of a matrix in a linear system. A permutation matrix is occasionally included in these products. The following steps as LU Decomposition.

Consider the set of linear equations.C_{11} x_1+C_{12} x_2+C_{13} x_3=d_1 $C_{21}x_1$++$C_{22}x_2$+C_{23} x_3=d_2 $C_{31}x_1$++$C_{32}x_2$+$C_{33}x_3$=d_3

The System is Equivalent to AX=B

Where A= $\begin{matrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{matrix}$ x=$\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}$ B=$\begin{matrix} d_1 \\ d_2 \\ d_3 \end{matrix}$

We factorize a as the product of lower and upper triangular Matrix.

l = $\begin{matrix} 1 & 0 & 0 \\ L21 & 1 & 0 \\ L31 & L32 & 1 \end{matrix}$ & u= $\begin{matrix} U11 & U12 & U13 \\ 0 & U22 & U23 \\ 0 & U & U33 \end{matrix}$

luX=B, uX=Y & lY=B

ie)$\begin{pmatrix} 1 & 0 & 0 \\ L21 & 1 & 0 \\ L31 & L32 & 1 \end{pmatrix}$ $\begin{matrix} y1 \\ y2 \\ y3 \end{matrix}$ = $\begin{matrix} d_1 \\ d_2 \\ d_3 \end{matrix}$

y1=d_1, L21y1+y2=d_2, L31y1+L32 y2+y3= d_3

$\begin{matrix} U11 & U12 & U13 \\ 0 & U22 & U23 \\ 0 & 0 & U33 \end{matrix}$ $\begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix}$ = $\begin{matrix} y1 \\ y2 \\ y3 \end{matrix}$

Ie. U11x_1+U12x_2+U133=Y1, U22x_2+U23x_3=Y2, U33x_3=Y3

from x_1,x_2, x_3 can be solved by back substation ,since y1,y2,y3 are known if u is known.

Now l and u can be found from lu=A

L21= $\frac{c21}{c11}$ U22= C22-$\frac{c21}{c11}$ .c12, U23=C23-$\frac{c21}{c11}$ .c13, L3= $\frac{c31}{c11}$ L32= $\frac{c32-c31.c12}{c22-\frac{c21}{c11}.c12}$, U33=C23-$\frac{c21}{c11}$ $c13$

Therefore l & u are Known using phython code below with one example.

LU Decompositon Phython code

In this subsection, we presented how to coding phython programme in LU Decomposition method

```
import pprint
import scipy
import scipy.linalg   # SciPy Linear Algebra
Library

A = scipy.array([ [7, 3, -1, 2], [3, 8, 1, -4], [-1, 1,
4, -1], [2, -4, -1, 6] ])
P, L, U = scipy.linalg.lu(A)

print "A:"
pprint.pprint(A)

print "P:"
pprint.pprint(P)

print "L:"
pprint.pprint(L)

print "U:"
pprint.pprint(U) A:
array([[ 7,  3, -1,  2],
       [ 3,  8,  1, -4],
       [-1,  1,  4, -1],
       [ 2, -4, -1,  6]])
P:
array([[ 1.,  0.,  0.,  0.],
       [ 0.,  1.,  0.,  0.],
       [ 0.,  0.,  1.,  0.],
       [ 0.,  0.,  0.,  1.]])
L:
array([[ 1.        ,  0.        ,  0.        ,  0.        ],
       [ 0.42857143,  1.        ,  0.        ,  0.        ],
       [-0.14285714,  0.21276596,  1.        ,  0.
],
       [ 0.28571429, -0.72340426,  0.08982036,
1.        ]])
U:
array([[ 7.        ,  3.        , -1.        ,  2.        ],
       [ 0.        ,  6.71428571,  1.42857143, -
4.85714286],
       [ 0.        ,  0.        ,  3.55319149,
0.31914894],
       [ 0.        ,  0.        ,  0.        ,  1.88622754]])
[0.2857142857142857, -0.7234042553191489,
0.0898203592814371, 1.0]]
U:
[[7.0, 3.0, -1.0, 2.0],
 [0.0, 6.714285714285714,
1.4285714285714286, -4.857142857142857],
 [0.0, 0.0, 3.5531914893617023,
0.31914893617021267],
 [0.0, 0.0, 0.0, 1.88622754491018]]
```

Numerical Example

For one type of milk, you need (0,1,2,) $(4,5,6)(-1,1,3)$of water, for another, (0,2,,4) , $(0,1,2)$ $(2,3,4)$ of water, and for the last one, (1,3,5), (-1,1,3) , $(2,4,6)$ of water. Assuming that there is no shortage of the other items needed to make the milk, calculate how many liters of milk can be manufactured with (12,14,16) ,(12,13,14) ,(16,17,18) of fat .Solve by Traingularization Method.

Solution

Consider the following linear system can be written as

(0, 1, 2,) x_1+(4,5,6) x_1+(-1,1,3) x_1=(12,14,16)

(0, 2, 4) x_1+(0,1,2) x_1+(2,3,4) x_1=(12,13,14)

(1,3,5) x_1+(-1,1,3) x_1+(2,4,6) x_1=(16,17,18)

This is equivalent to converting the above triangular fuzzy number to crisp number by the average ranking function

$$\begin{bmatrix}1 & 5 & 1\\2 & 1 & 3\\3 & 1 & 4\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}14\\13\\17\end{bmatrix}$$

$AX = B$

For lu $\begin{bmatrix}1 & 0 & 0\\L21 & 1 & 0\\L31 & L32 & 1\end{bmatrix}\begin{bmatrix}U11 & U12 & U13\\0 & U22 & U23\\0 & 0 & U33\end{bmatrix} = \begin{bmatrix}1 & 5 & 1\\2 & 1 & 3\\3 & 1 & 4\end{bmatrix}$

He can write U11=1,U12=5,U13=1

$$\begin{bmatrix}1 & 0 & 0\\L21 & 1 & 0\\L31 & l32 & 1\end{bmatrix}\begin{bmatrix}1 & 5 & 1\\0 & U22 & U23\\0 & 0 & U33\end{bmatrix}\begin{bmatrix}1 & 5 & 1\\2 & 1 & 3\\3 & 1 & 4\end{bmatrix}$$

$hence$ L21=2,5L21+U22=1,L21+U23=3

L21=2,U22=9,U23=1, Again
L31=3,5L31+L32U22=1, L31+L32U23+U33=4,
L32=$\frac{14}{9}$,U33=$\frac{-5}{9}$, lY=6 gives

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & \frac{14}{9} & 1 \end{bmatrix}\begin{bmatrix} y1 \\ y2 \\ y3 \end{bmatrix}=\begin{bmatrix} 14 \\ 13 \\ 17 \end{bmatrix}$$

Ie y1=14 , 2y1+y2=13, 3y1+$\frac{14}{9}$ y2+y3=17

Y1=14, y2=15, y3=$\frac{-5}{3}$, ux=y implies

$$\begin{bmatrix} 1 & 5 & 1 \\ 0 & 9 & 1 \\ 0 & 0 & \frac{-5}{9} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 14 \\ -15 \\ \frac{-5}{3} \end{bmatrix}$$

ie x_1+5x_2+x_3=14, -9x_2+x_3=-15, $\frac{-5}{9}$ x_3=$\frac{-5}{3}$
finally $x_1 = 1$ $x_2 = 2$ $x_3 = 3$.

Based on the numarical value (Percentage), we get which product given to Childhood, Adult and Senior citizens. This is very useful to growth our health to maintain average level of fat contents.

Conclusion

In this article, a set of milk products that are deemed to be triangular fuzzy data are based on the definition of fat. We suggested a strategy to identify the best, average, and good products from those rated on a numerical scale for taken age factros. Finally, a concrete example is provided to support the suggestion. This method is used to resolve linear systems and find the inverse matrix. Computer software is also favored to use the factorization process.

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